before arriving at the usually observed diamond buckling pattern. His plans for further and better studies of this type may clear up some of the unanswered questions about the buckling process for cylinders in axial compression. In fact, the experimental determination of the true "buckling mode" may well require the use of this technique, since it is possible that the shell configuration undergoes significant changes after the inception of buckling and before deformations become visible to the eye or camera.

However, the author claims to have attained experimental buckling loads within 10% of the classical load and, on this basis, concludes that the classical load is attainable if the cylinder is free of initial imperfections. These statements deserve careful scrutiny. In the first place, it is evident from the different slopes of the theoretical and experimental curves of the author's Fig. 1 that the true extensional stiffness of the experimental cylinder material is somewhat greater than that used in the calculation of the classical load. If the experimental result is compared with the correct classical load for the cylinder specimen, the discrepancy between theory and experiment indicated in the author's figure is more than doubled to 16%; presumably, this figure is typical of the author's results. Secondly, the author's cylinder specimen has a relatively low ratio of radius to thickness ($R/t = 154$), and it is well known that buckling loads as high as the author's are attainable for such cylinders without unusual efforts to eliminate shape imperfections. For example, Fig. 1 is a plot of buckling load against R/t with the author's result shown by the solid circle. The hatched area encompasses a large body of data taken from the work of 14 different investigators and compiled in Ref. 2. These experiments were conducted under widely varying conditions on specimens, in the range $0.5 < L/R < 5$, made from a variety of materials. The open circles are additional recent results[3] obtained on 7075-T6 aluminum alloy cylinders with $L/R = 4$. Normal production methods were used in manufacturing these cylinders. The wide variation in the results shown in Fig. 1 may well be due partly to initial imperfections; however, the spread probably also is caused, in part, by variations in testing conditions. The salient fact is that the author's result does not constitute new or unique evidence of the importance of imperfections and certainly does not warrant his strong conclusion.

More convincing evidence of the importance of careful manufacture and testing is provided by the results[4] shown by the square symbols in Fig. 1. These were obtained from tests of copper cylinders with $L/R = 2.5$ which were made by carefully electroplating the shells on accurately machined wax mandrels. Radius variations in these shells were held to within half the thickness and thickness variations to 3%. The tests were conducted carefully with special attention to alignment. In contrast to the author's result, these results are seen to represent a clear and significant improvement over the general body of existing data. Moreover, additional tests (see Ref. 4) of cylinders with specified imperfections under carefully maintained uniform conditions have confirmed that imperfections alone can indeed cause decreases in strength of up to 40%.

However, even the results of Ref. 4 are not sufficient evidence to support the author's conclusion, for the author is completely disregarding an important source of error in the classical theory which is entirely unrelated to initial shape imperfections: the inconsistent assumption made in classical theory regarding end conditions. Whereas in classical theory it is tacitly assumed that the ends are free prior to buckling so that the cylinder retains its shape and uniform (membrane) stress up to the buckling load, the ends of a real cylinder actually are restrained (as in the author's experiments, for example) so that the prebuckling shape and stress state is the axisymmetric state calculated by Föppl.[5] Stein[6] has shown that axial compression buckling (in the classical sense) away from the Föppl prebuckling state may occur at

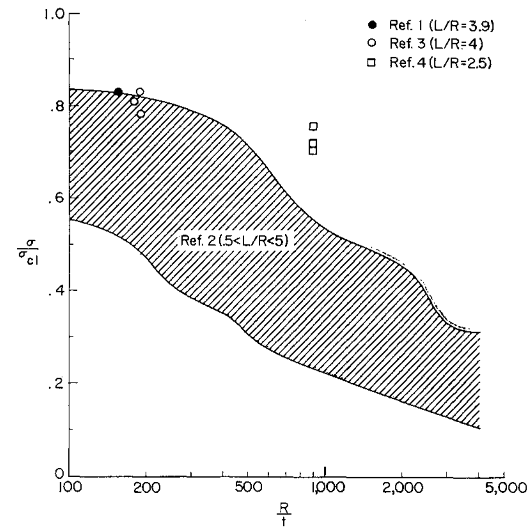

Fig. 1 Influence of radius-thickness ratio on buckling data for nominally perfect cylinders in axial compression.

about half the classical buckling load for one case of simply supported, initially perfect cylinders. Presumably, the reduction would be less for cylinders with clamped ends, but similar solutions for clamped cylinders are not yet available. The author's test specimen probably approximates a clamped case, but it appears highly unlikely that he will be able to achieve the classical buckling load merely by eliminating initial shape imperfections from his experimental cylinders.

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Reply by Author to R. W. Leonard

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The experimental results in Fig. 1 of the author's note[1] were plotted using the strain readings determined from photoelastic data taken at one location on the shell. The actual value of Young's modulus used in calculating the critical buckling load of the shell was that of the shell deter-

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mined experimentally. The scatter gives a measure of the accuracy of the photoelastic technique in determining strain and also indicates the variation of stress in the shell. The usual procedure is to plot stress ratio (σ/σ_{cr}) vs end deformation of the shell. However, the author simply wanted to illustrate a typical test run using the photoelastic method of computing strain.

Recent results from several photoelastic shells tested by the author have confirmed the fact that buckling loads within 10% of the classical value are obtainable. Although the shells employed had relatively low ratios of radius to thickness (i.e., $100 \leq R/t \leq 170$), the buckling loads represent an increase (about 10%) over the data in Fig. 1 of the preceding comment for the same value of R/t. The author admits that the results of Ref. 2 represent a "significant improvement over the general body of existing data" because of the higher R/t ratio of the shells tested.

In the concluding remarks of Ref. 1, the author wished to point out the following:

1) Of the two existing competitive theories explaining the lower buckling loads (well below the classical value), Tsien's energy criterion, as well as having no logical basis,[3] appeared to have no experimental basis. Tsien's lower buckling load, as stated by the author,[1] was achieved only by applying a lateral disturbance to the shell.

2) When isotropic elastic shells are made sufficiently free of imperfections and loaded axially in a rigid test machine, buckling loads very close to the classical value can be achieved. The author never intended his note to constitute unique evidence of the importance of imperfections on the buckling load of a shell loaded axially in compression. Furthermore, his *tentative* conclusion was proposed knowing full well that further investigation of both end effects and imperfections on the shell buckling phenomena was necessary.

The author recognizes the fact that the classically predicted critical buckling load of a circular cylindrical shell can be attained only within certain limits. He also accepts the fact that classical theoretical solutions have assumed shell edges that are free to expand radially, and, since his cylinder edges were clamped, this posed a limit on the observed buckling loads. The theoretical analysis of Ref. 4 for the case of simply supported ends clearly indicates that the end conditions do play a role in reducing the critical buckling stress of a shell. However, end constraints did not appear to produce a serious effect on the clamped shells used by the author. Buckling loads were repeatable and always near 10% of the computed value. No appreciable end effect has been observed by the author in the photoelastic analysis of the shells, either in the prebuckled state or in the buckled configuration. Any bending deformations should appear as color striations at the ends of the loaded shell.

In conclusion, the author agrees with Leonard that very convincing experimental evidence of the effect of imperfections on the lowering of buckling loads is offered by the results of Ref. 2. However, it must be noted that the works of both Refs. 2 and 4 did not appear until December 1962, after the author had submitted his note (August 1962) for publication.

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Comment on "Flight Mechanics of the 24-Hour Satellite"

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In his article on the flight mechanics of the 24-hr satellite, Perkins[1] refers to an unpublished manuscript by Blitzer, Boughton, Kang, and Page[2] as having "shown that the rate of longitudinal drift of the 24-hr equatorial satellite due to equatorial oblateness is sufficiently large to be of concern to system designers." He further states that "With the exception of the period for the case of small amplitude oscillation, Blitzer's work did not present any closed-form analytical expressions for the mean path motion of the satellite."

The object of this note is to update Perkins' reference and inform the reader of two papers by Blitzer et al.[3, 4] which appeared prior to Perkins' article and in which the restrictions to equatorial orbits and small-amplitude longitudinal oscillations were removed. In the latter paper,[4] particularly, closed-form analytic expressions are given for the period and for both radial and longitudinal motions. Diurnal and long-period components are included in the expressions. The solution for the mean motion was motivated by noting the similarity of the problem to that of the physical pendulum.

Perkins adopts an independent approach to the problem, but our results appear to be consistent. The reader will also find some other recent papers[5, 6] to be of relevance.

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Comment on "Dynamic Analysis for Lunar Alightment"

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In a recent paper by Cappelli,[1] an analytical procedure was described for obtaining the motion during touchdown of a spacecraft landing on the lunar surface. Comparison of some

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